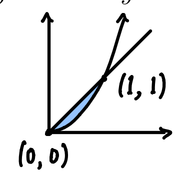
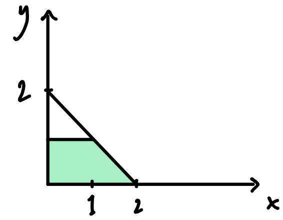
1. Let $\mathcal{D}$ be the region in the first quadrant of the plane bounded by the curves $y = x$ and $y = x^3$. Write the integral $\int\int_{\mathcal{D}} f(x, y)\, \mathrm{d}A$ in the two possible orders.



$$\int_0^1 \int_{x^3}^{x} f(x, y)\, dy\, dx, \quad \int_0^1 \int_{y}^{y^{1/3}} f(x, y)\, dx\, dy$$

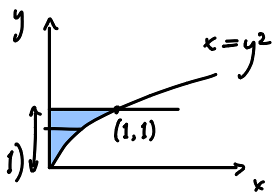
2. Let $\mathcal{D}$ be the region in the plane bounded by the lines $y = 0$, $x = 0$, $y = 1$, and $y = -x + 2$. Write the double integral $\int\int_{\mathcal{D}} f(x, y)\, \mathrm{d}A$ as an iterated integral in both possible orders. Which is probably going to be less work to compute?



$$\int_0^2 \int_0^{\min(1, -x+2)} f\, dy\, dx = \int_0^1 \int_0^1 f\, dy\, dx + \int_1^2 \int_0^{2-x} f\, dy\, dx$$

$$\int_0^1 \int_0^{-y+2} f\, dx\, dy$$ ⟵ would probably want to do this one

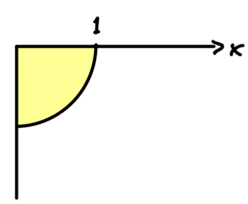
3. Compute the double integral $\int\int_{\mathcal{D}} \sqrt{y^3 + 1}\, \mathrm{d}A$ where $\mathcal{D}$ is the region of the first quadrant bounded by $y = 1$ and $y = \sqrt{x}$. Try to compute it in both orders– is one way easier than the other?



$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\, dy\, dx, \qquad (y^3+1)^{3/2}$$

$$\int_0^1 \int_0^{y^2} \sqrt{y^3+1}\, dx\, dy = \int_0^1 y^2 \sqrt{y^3+1}\, dy = \frac{2}{9}(y^3+1)^{3/2}\Big|_0^1 = \frac{2}{9}\left(2\sqrt{2} - 1\right)$$

4. Compute the integral $\int_0^1 \int_{-\sqrt{1-x^2}}^{0} 2x \cos(y - y^3/3)\, \mathrm{d}y\, \mathrm{d}x$.



$$= \int_{-1}^0 \int_0^{\sqrt{1-y^2}} 2x \cos(y - y^3/3)\, dx\, dy = \int_{-1}^0 (1-y^2)\cos(y - y^3/3)\, dy$$

$$= \sin(y - y^3/3)\Big|_{-1}^0 = \sin\frac{2}{3}$$

5. In this problem you'll compute the triple integral $\int\int\int_{\mathcal{W}} y\, \mathrm{d}V$ where $\mathcal{W}$ is the volume bounded between the paraboloids $z = x^2 + y^2$ and $z = 4 - (x^2 + y^2)$.

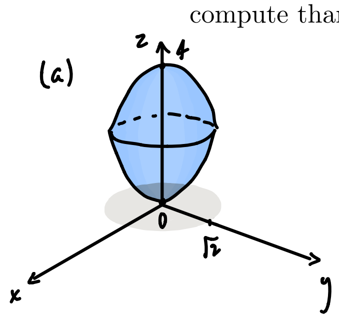
   (a) Draw a sketch of this region.

   (b) Observe that this region is z-simple and compute the projection of the region in the xy-plane.

   (c) Write the integral $\int\int\int_{\mathcal{W}} y\, \mathrm{d}V$ as an iterated integral. Since you described the region as z-simple you will write this integral as $\mathrm{d}z\, \mathrm{d}x\, \mathrm{d}y$ or $\mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x$.

   (d) Compute the integral $\int\int\int_{\mathcal{W}} y\, \mathrm{d}V$. One of the two ways to set up this integral is much easier to compute than the other– why?

(a)



(b) projection is the disc $\{(x, y) : x^2 + y^2 \leq 2\}$, since the boundary consists of $(x, y) : x^2 + y^2 = 4 - x^2 + y^2$.

(c)

$$\iiint_W y \, dV = \iint_{x^2+y^2 \le 2} \int_{x^2+y^2}^{4-(x^2+y^2)} y \, dz \, dA$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{x^2+y^2}^{4-(x^2+y^2)} y \, dz \, dy \, dx \qquad (\text{resp. } x \leftrightarrow y)$$

(d)

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \underbrace{(4 - 2(x^2+y^2))y}_{(4-2x^2)y \, - \, 2y^3} \, dy \, dx$$

interval is symmetric about 0

odd in y

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left((2 - x^2)y^2 - \frac{1}{2} y^4\right) \Big|_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}} dx = 0$$