

MATH 32B SPRING 20 WEEK 1 WORKSHEET

1. Introduce yourself to your group, write down their names, and find three things that you and your group members share in common. Consider exchanging contact information so you can study together—this quarter more than ever it will be helpful to find study buddies.

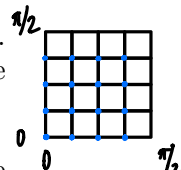
2. Consider the surface defined by $z = \sin(x + y)$ over the rectangle $\mathcal{R} = [0, \pi/2] \times [0, \pi/2]$. Use a double Riemann sum with $m = n = 4$ to approximate the volume under the surface using lower left corners as sample points and upper right quarters as sample points.

$$\sum_{i,j=0,0}^{4,4} \left(\frac{\pi}{8}\right)^2 \sin\left((i+j)\frac{\pi}{4}\right)$$

actual value: 2

≈ 1.87169

≈ 1.87169



3. Let $\mathcal{R} = [a, b] \times [c, d]$ be a rectangle in the plane. Find $\int \int_{\mathcal{R}} 1 dA$ and $\int \int_{\mathcal{R}} k dA$ where $k \in \mathbb{R}$ is a constant.

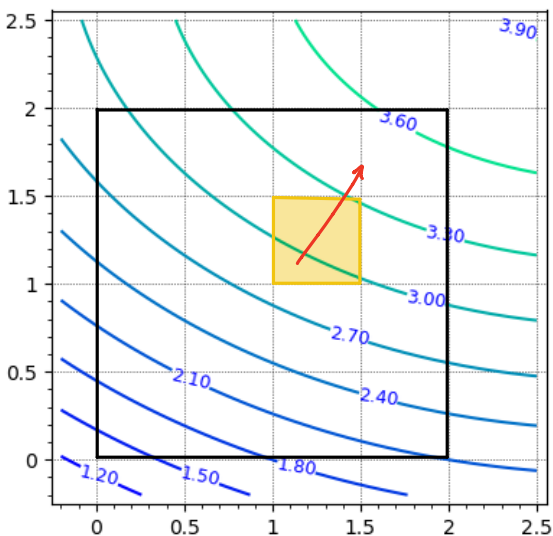
$= \text{vol } \mathcal{R} \qquad = k \text{ vol } \mathcal{R}$

4. Based on your answer to the previous question, if \mathcal{R} is any shape in the plane what is $\int \int_{\mathcal{R}} 1 dA$? $\text{vol } \mathcal{R}$ (subject to some restrictions on the shape \mathcal{R})

5. Consider the rectangle $\mathcal{R} = [0, 1] \times [-1, 1]$. For which of the following functions is $\int \int_{\mathcal{R}} f(x, y) dA = 0$?

- (1) $f(x, y) = e^{x^2+y^2} x > 0$
- (2) $f(x, y) = \cos(x + y) \sin(xy) \approx -0.1712$
- ③ $f(x, y) = \cos(xy) \sin(xy)$ since odd in y
- (4) $f(x, y) = xy^2 > 0$

6. Consider the following contour plot of a function $h(x, y)$.



Note that f is increasing (locally) in the direction of the red arrow. So we can see f is bounded below by 2.7 & above by 3.6 on the gold square.

Use Riemann sums with $m = n = 4$ to estimate $\int \int_{[0,2] \times [0,2]} h(x, y) dA$. Give an upper and a lower bound for $\int \int_{[0,2] \times [0,2]} h(x, y) dA$.

lower $\left(\frac{1}{2}\right)^2 (1.2 + 1.5 + 1.8 + 1.8 + 1.8 + 2.1 + 2.1 + 2.4 + 2.1 + 2.4 + 2.7 + 2.7 + 2.4 + 2.7 + 3 + 3.3) = 9$

upper $\left(\frac{1}{2}\right)^2 (2.1 + 2.4 + 2.7 + 2.7 + 2.7 + 3 + 3 + 3.3 + 3 + 3.3 + 3.6 + 3.6 + 3.3 + 3.6 + 3.6 + 3.9) = 12.45$