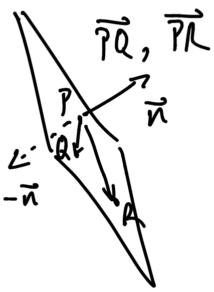
WORKSHEET
DISCUSSION SECTION 3
DUE 4/22 AT MIDNIGHT PDT

P Q R

(1) Let P be the plane that contains the points $(1,0,0)$, $(0,2,0)$, and $(0,0,3)$.
  (a) Find two vectors $\vec{u}$ and $\vec{v}$ that are tangent (parallel) to P.
  (b) Use the cross product to find a normal (perpendicular) vector to P.
  (c) Find an equation for P.
  (d) Is the point $(1,2,3)$ in P? Why or why not?



(2) Let ℓ be the line parametrized by

$$\langle 1,2,3\rangle + s\langle 4,5,6\rangle$$

and let P be the plane with equation

$$x + 2y + 3z = -18.$$

Do the line ℓ and the plane P intersect? If so, find the point where they intersect. If not, explain why.

(3) Let ℓ be the line parametrized by

$$\langle 1,2,3\rangle + s\langle 4,5,6\rangle$$

and let m be the line parametrized by

$$\langle 14,17,20\rangle + t\langle 1,1,1\rangle.$$

Do the lines ℓ and m intersect? If so, find the point where they intersect. If not, explain why.

(4) Consider the elliptic cone

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \left(\frac{z}{c}\right)^2.$$

in 3D space. Describe all of its xy, xz, and yz traces.

We need $a, b, c \neq 0$.
We may take $a, b, c > 0$
without changing the eqn.

(5) Consider the cylinder

$$y = \sin(x)$$

in 3D space. Describe all of its xy, xz, and yz traces.

1a. $\vec{u} = (0,2,0) - (1,0,0) = (-1,2,0)$ $\vec{v} = (0,0,3) - (1,0,0) = (-1,0,3)$

b. $$\vec{n} = \vec{u} \times \vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}\hat{\imath} - \begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix}\hat{k}$$
$$= (6,3,2)$$

c. $(6,3,2)\cdot(x,y,z) = (6,3,2)\cdot(1,0,0) = 6$
$6x + 3y + 2z = 6$

d. No, $6\cdot 1 + 3\cdot 2 + 2\cdot 3 = 18 \neq 6$

2. $(1+4s,\ 2+5s,\ 3+6s)$

$x+2y+3z=-18$

$$(1+4s)+2(2+5s)+3(3+6s) = -18$$

$$\| $$

$$1+4+9+4s+10s+18s$$

$$\| $$

$$14+32s$$

$$\Rightarrow\ 32s=-32\ \Rightarrow\ s=-1$$

$\Rightarrow$ they intersect at $(1+4(-1),\ 2+5(-1),\ 3+6(-1))$

$= (-3,-3,-3)$

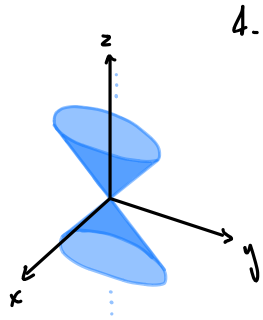
3. If there exist $s,t\in\mathbb{R}$ for which

$(1,2,3)+s(4,5,6) = (14,17,20)+t(1,1,1)$:

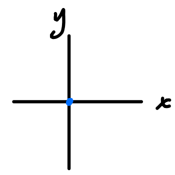
$$\left.\begin{aligned} 1+4s &= 14+t \\ 2+5s &= 17+t \end{aligned}\right\} \Rightarrow\ 1+4s-14 = 2+5s-17\ \Rightarrow\ s=2,\ t=1+4(2)-14=-5$$

$$3+6s = 20+t \qquad 3+6\cdot 2 = 15 = 20+(-5)$$

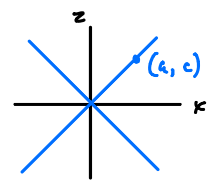
so these lines intersect at $(9,12,15)$



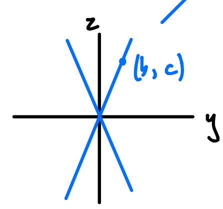
4. xy: $\ z=0\ \Rightarrow\ x^2/a^2 + y^2/b^2 = 0\ \Rightarrow\ x=y=0$

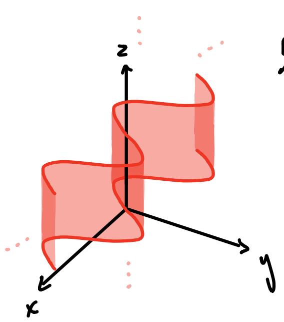


xz: $\ y=0\ \Rightarrow\ x^2/a^2 = z^2/c^2\ \Rightarrow\ z=\pm\frac{c}{a}|x|$

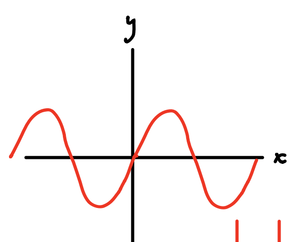


yz: $\ x=0\ \Rightarrow\ y^2/b^2 = z^2/c^2\ \Rightarrow\ z=\pm\frac{c}{b}|y|$



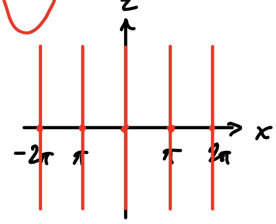


5. $y=\sin x$ $\qquad$ xy: $\ z=0\ \Rightarrow\ y=\sin x$



xz: $\ y=0\ \Rightarrow\ \sin x = 0\ \Rightarrow\ x=\pi k\ \ (k\in\mathbb{Z})$

z arbitrary



yz: $\ x=0\ \Rightarrow\ y=0,\ z$ arbitrary