WORKSHEET
DISCUSSION SECTION 1
DUE 4/5 AT MIDNIGHT

(1) Compute the following derivatives.
 (a) Compute $\frac{d}{dx}x^n$ using the power rule.
 (b) Compute $\frac{d}{dx}(x^{n-1}\cdot x)$ using the product rule and the power rule for x^{n-1}.
 (c) Compute $\frac{d}{dx}(x^{n+1}/x)$ using the quotient rule and the power rule for x^{n+1}.
 (d) Compute $\frac{d}{dx}a^x$, where $a^x = e^{x\ln a}$.
 (e) Compute $\frac{d}{dx}x^x$, where $x^x = e^{x\ln x}$.

(a) nx^{n-1}

(b) $(n-1)\,x^{n-2}\cdot x + x^{n-1}\cdot 1 = nx^{n-1}$

(c) $\dfrac{x\cdot(n+1)\,x^{n} - x^{n+1}\cdot 1}{x^2} = \dfrac{nx^{n+1}}{x^2} = nx^{n-1}$

(d) $\frac{d}{dx}a^x = e^{x\ln a}(\ln a) = a^x\ln a$

(e) $\frac{d}{dx}x^x = e^{x\ln x}\left(1\ln x + x\left(\frac{1}{x}\right)\right) = x^x(\ln x + 1)$

(2) Consider the function $f : [0,1] \to \mathbb{R}$, defined by $f(x) = x + x^2 + x^3 + x^4$.
 (a) Explain why $f(a) = 2$ for some $0 < a < 1$.
 (b) Give two reasons why $f'(b) = 4$ for some $0 < b < 1$.
 Hint: use the intermediate value theorem and mean value theorem.

(a) f is continuous on $[0, 1]$ and $f(0) = 0$, $f(1) = 4$. Since $2 \in (0, 4)$, the intermediate value thm. yields an $a \in (0, 1)$ s.t. $f(a) = 2$.

(b) 1. $f' = \underline{1 + 2x + 3x^2 + 4x^3}$ so $f'(0) = 1$, $f'(1) = 10$. Since f' is continuous, identical reasoning to the above shows that there is a $b \in (0, 1)$ s.t. $f'(b) = 4$.

2. f is continuous on $[0, 1]$ and differentiable in $(0, 1)$, so by the mean value thm., there is a $b \in (0, 1)$ s.t.

$$f'(b) = \frac{f(1) - f(0)}{1 - 0} = \frac{4 - 0}{1 - 0} = 4.$$

(3) A *complex number* is any number of the form $z = x + iy$, where x and y are real numbers, and $i = \sqrt{-1}$. We write $x + iy = a + ib$ if $x = a$ and $y = b$.
  (a) Find the complex solutions of $x^2 + 4 = 0$
  (b) Find the complex solutions of $x^2 + 2x + 2 = 0$.
  Euler's formula is: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. It follows from comparing power series.
  (c) Show that $e^{i\pi} = -1$.
  (d) Expand both sides of $e^{i(x+y)} = e^{ix} \cdot e^{iy}$ to derive the formulas

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \quad \text{and} \quad \sin(x+y) = \cos(x)\sin(y) + \sin(x)\cos(y).$$

This is one way to re-derive trig identities on the fly. (I never remember them.)

(a) $x = \pm 2i$

(b) $x^2 + 2x + 2 = 0 \Rightarrow x \overset{\text{by the quadratic formula}}{=} \dfrac{-2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = -1 \pm \frac{1}{2}\sqrt{-4}$

$= -1 \pm i$

(c) $e^{i\pi} = \cos\pi + i\sin\pi = -1 + 0i = -1$

(d) $e^{ix}e^{iy} = (\cos x + i\sin x)(\cos y + i\sin y)$

$= (\cos x\cos y - \sin x\sin y) + i(\sin x\cos y + \cos x\sin y)$

& $e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$

so matching real & imaginary parts gives

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

as desired.