WORKSHEET
DISCUSSION SECTION 4
DUE 4/29 AT MIDNIGHT PDT

(1) Consider the quadric surface S with equation

$$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = \left(\frac{z}{4}\right)^2 + 1.$$

$$\left(\frac{y}{3}\right)^2 - \left(\frac{z}{4}\right)^2 = 1 - \left(\frac{h}{2}\right)^2$$

(a) What kind of surface is S?

see next page

(b) Describe all of the yz-traces of S, and indicate where the geometry changes. In other words, describe the conic sections obtained by intersecting S with each of the planes $x = h$, and identify the values of h where these conics change. $\longrightarrow$ at $h = \pm 2$

(2) Consider the helix

$$\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle.$$

(a) Sketch $\vec{r}(t)$.

(b) Compute the velocity, speed, and acceleration of $\vec{r}(t)$. Draw some tangent vectors into your picture from (a).

(c) Let $\vec{s}(t) = \langle -\cos(t), -\sin(t), t \rangle$. What sort of shape do the trajectories of $\vec{r}$ and $\vec{s}$ form together?

(d) **(Optional)** Parametrize the line segment connecting $\vec{r}(n)$ and $\vec{s}(n)$, for any integer n. What do these line segments and the trajectories of $\vec{r}$ and $\vec{s}$ form?

(3) Consider the following parametrization of an Archimedean spiral

$$\vec{r}(t) = \langle t\cos(t), t\sin(t) \rangle.$$

$$\vec{r}\,'(t) = (\cos t - t\sin t,\ \sin t + t\cos t)$$

$$\vec{r} \cdot \vec{r}\,'(t) = t\cos^2 t - \cancel{t^2 \cos t \sin t} + t\sin^2 t + \cancel{t^2 \sin t \cos t} = t$$

(a) Sketch the trajectory of $\vec{r}$ for $t \geq 0$.

(b) Parametrize the tangent line to the trajectory of $\vec{r}$ at $\vec{r}(a)$.

(c) Compute the angle $\theta(a)$ between $\vec{r}(a)$ and $\frac{d\vec{r}}{dt}(a)$.

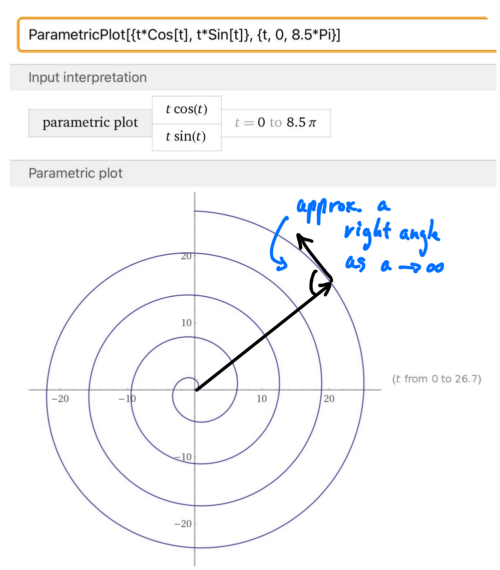
(d) Does the limit

$$\lim_{a \to \infty} \theta(a)$$

exist? If so, what is it? If not, explain why.

(e) **(Optional)** What does your answer in (c) tell you about the tangent line to the Archimedean spiral? How does this compare to the tangent lines of a circle? To the tangent lines of a logarithmic spiral?

3a.



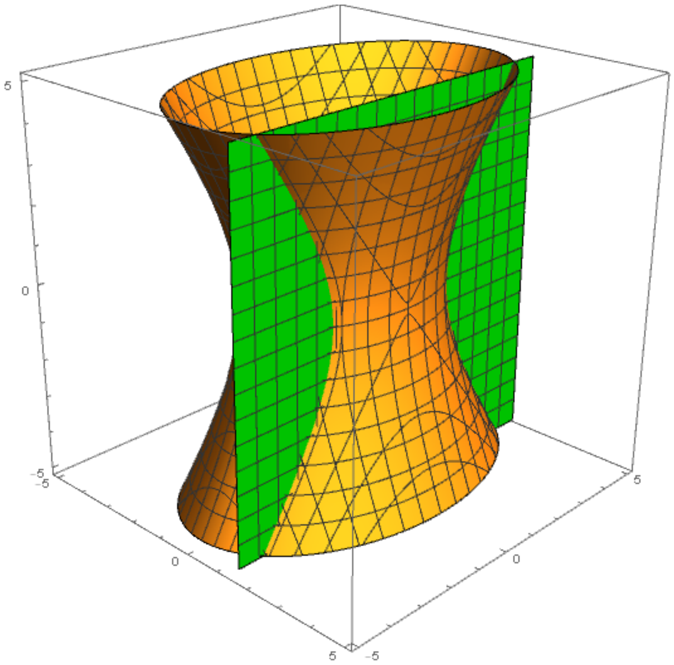
3b. $$\vec{L}(s) = \vec{r}(a) + \vec{r}\,'(a)s$$
$$= (a\cos a,\ a\sin a) + (\cos a - a\sin a,\ \sin a + a\cos a)s \quad (s \in \mathbb{R})$$

c. $$\|\vec{r}\,'(t)\|^2 = (\cos t - t\sin t)^2 + (\sin t + t\cos t)^2 = 1 + t^2$$

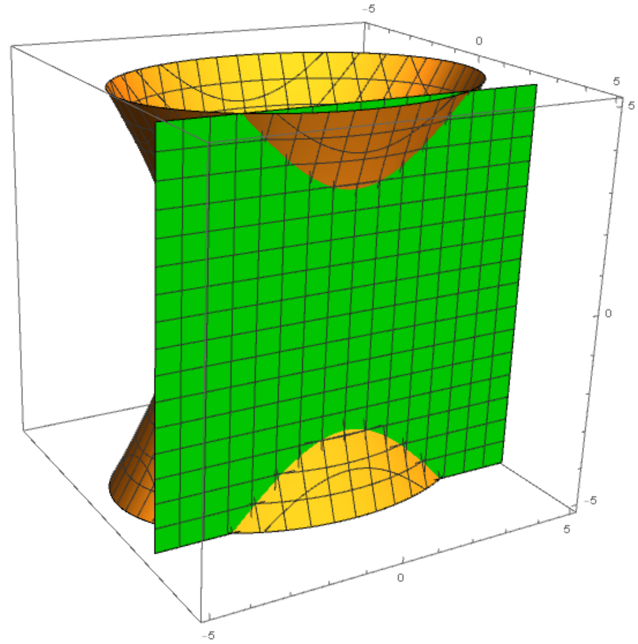
$$\theta(a) = \arccos \frac{\vec{r} \cdot \vec{r}\,'(a)}{\|\vec{r}(a)\| \|\vec{r}\,'(a)\|}$$

$$= \arccos \frac{1}{\sqrt{1+a^2}} \xrightarrow{d.} \frac{\pi}{2} \text{ as } a \to \infty$$

e. it becomes orthogonal as $t \to \infty$.



hyperboloid with $x = 1.5$



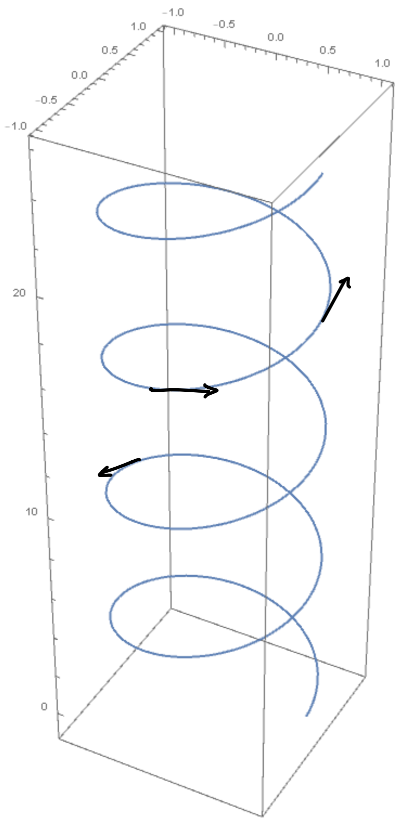
hyperboloid with $x = 2.5$

Traces for $x = h$:

$|h| < 2$: hyperbola with major axis y

$|h| > 2$: hyperbola with major axis z

$|h| = 2$: X $\left(\text{i.e. } \left(\frac{y}{3}\right)^2 = \left(\frac{z}{4}\right)^2\right)$



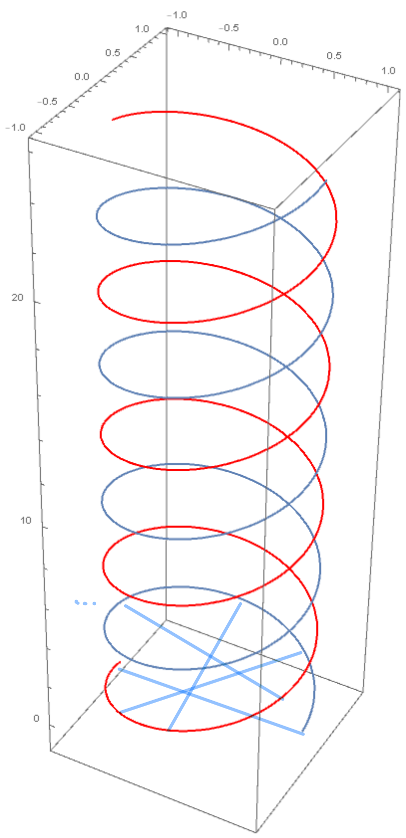
2. $\vec{v}(t) = \vec{r}\,'(t) = (-\sin t, \cos t, 1)$

$$v(t) = \|\vec{v}(t)\| = \sqrt{(-\sin t)^2 + \cos^2 t + 1^2} = \sqrt{2}$$

$$\vec{a}(t) = (-\cos t, -\sin t, 0)$$

<u>Note</u>: we can parameterize a line segment from $\vec{u}$ to $\vec{v}$ by

$$\vec{l}(t) = (1-t)\vec{u} + t\vec{v}, \quad 0 \le t \le 1.$$



With the segments, this might remind you of DNA