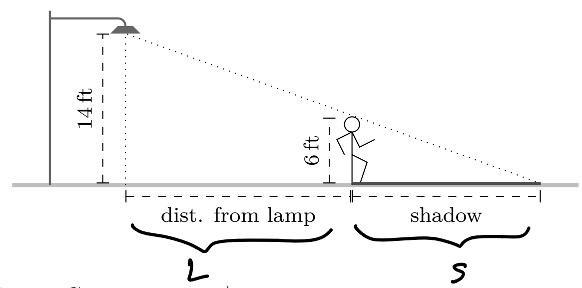
B. Stone

Math 31AL Worksheet
Thursday, Nov 7 (Week 6)

1. A woman whose height is 6 ft walks away from a street lamp at a speed of $3\,\frac{\text{ft}}{\text{s}}$. The street lamp is 14 ft high. Find the rate at which the woman's shadow is increasing in length.



(a) Identify two significant **variables**. *(Hint: See picture.)*

L, s as above

(b) What, in terms of the variables you listed in part (a), is **given** in the problem?

$\frac{dL}{dt} = 3$ ft/s

(c) What, in terms of the variables you listed in part (a), are you trying to **find**?

$\frac{ds}{dt}$

(d) Write down an **equation** relating the two variables.

since the triangles are similar $\rightarrow$ $\frac{L+s}{14} = \frac{s}{6} \Rightarrow \frac{L}{14} = \frac{s}{6} - \frac{s}{14} = \frac{2s}{21} \Rightarrow L = \frac{4}{3}s$

(e) Differentiate both sides of your equation from part (d), *with respect to t*. (Remember that you're thinking of the variables as *functions of t*.) Then plug in numbers and solve.

$L'(t) = \frac{4}{3}s'(t) \Rightarrow s'(t) = \frac{3}{4}\cdot 3 = 9/4$ ft/s

2. In this problem, you will use a linear approximation to approximate the value of $(1.1)^{4/3}$.

   (a) What function do you want to approximate here, and at what value of x?

   $$f(x) = x^{4/3} \qquad\qquad \text{at } x = 1.1$$

   (b) What's a number close to x where you know the value of this function? This number should be your "anchor point" a.

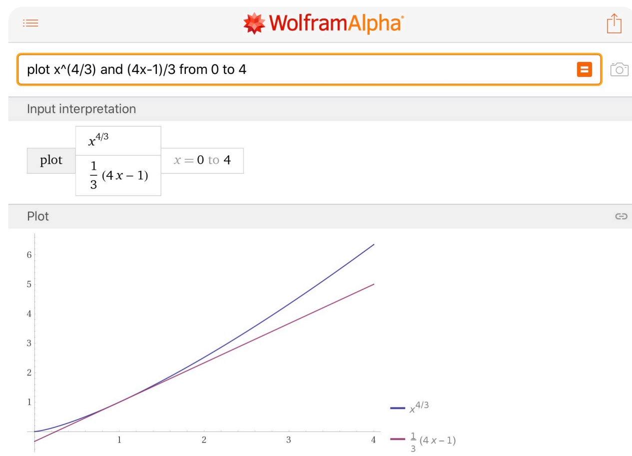
   $$a = 1$$

   (c) Find the equation of the tangent line to $f(x)$ at $x = a$.

$$f'(x) = \frac{4}{3}x^{1/3} \implies f'(1) = \frac{4}{3}$$

$$L(x) = f(a) + f'(a)(x-a) = f(1) + f'(1)(x-1)$$

$$= 1 + \frac{4}{3}(x-1) = \frac{4}{3}x - \frac{1}{3}$$



   (d) Plug the value of x from part (a) into your equation for the tangent line. This is the approximate value of $(1.1)^{4/3}$.

   $$L(1.1) = \frac{4}{3}\cdot 1.1 - \frac{1}{3} = \frac{3.4}{3} \approx 1.1333$$

   (e) All of the above could (in theory) be done without a calculator. Now use a calculator to compute the *actual* value of $(1.1)^{4/3}$. What is the error? What is the percent error?

$$(1.1)^{4/3} \approx 1.1355$$

$$\implies |err| \approx 1.1355 - 1.13333 \approx 0.00217$$

$$\%|err| \approx \frac{0.00217}{1.1355} \approx 0.191\,\%$$

(rel. to actual value)

3. Let $f(x) = \dfrac{1}{1+x^3}$.

(a) Suppose we start at $x = 2$ (so this is the "anchor point" a), and make a change in x of $\Delta x = 0.3$. Use the "short form" of the linear approximation formula,

$$\Delta f \approx f'(a) \cdot \Delta x,$$

to approximate the corresponding change in f (that is, Δf).

$$f'(x) = -(1+x^3)^{-2} \cdot 3x^2$$

$$\Delta f \approx f'(2) \cdot 0.3 = -(1+2^3)^{-2} \cdot 3 \cdot 4 \cdot 0.3$$
$$\approx -0.04445$$

(b) The *exact* value of Δf (that you approximated in part (a)) is a difference of two values of f. Using a calculator, compute this exact value of Δf. *(Hint: What two x values do you need to plug in?)*

$$\frac{1}{1+2.3^3} - \frac{1}{1+2^3} \approx -0.03516$$

(c) Compute the error and the percentage error between your approximation of Δf and the actual value.

$$|\text{err}| \approx 0.00929$$

$$\%\,|\text{err}| \approx 26.4\%$$

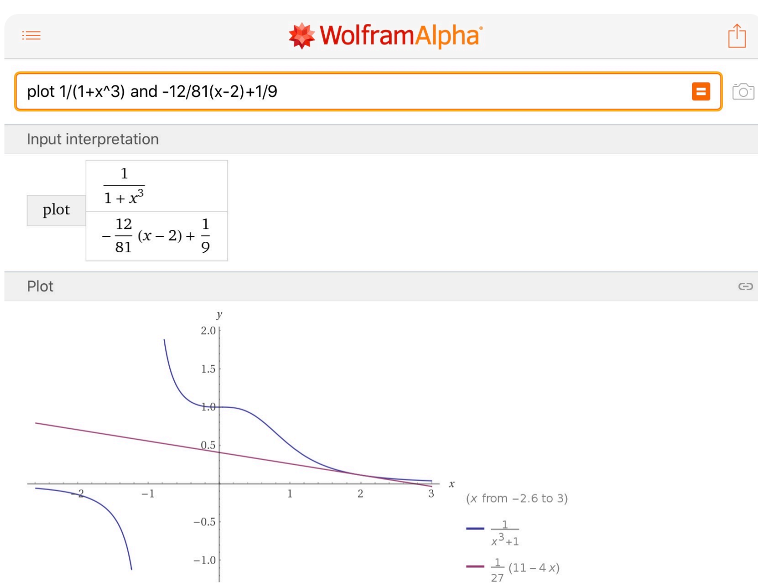


illustration of the tangent line

4. The revenue earned by a movie theater is

$$R(p) = 3200p - 8p^3$$

where p is the price of each movie ticket, in dollars. Currently, tickets cost \$12, but the theater is considering *raising the price* by \$0.50. In this problem, we will use a linear approximation to get a quick estimate of how much *more* (or *less*) revenue the theater will make if it raises the ticket price in this way.

(a) The two **variables** in this problem are p, the price of each ticket, and R, the amount of revenue the theater will earn. In terms of (one of) these variables, what does "raising the price by \$0.50" mean? (This is what you're **given** in the problem.)

increase p by $1/2$

(b) The problem asks you to **find** "how much more (or less) revenue...". In terms of the variables, what is this asking you to estimate?

$$\Delta R = R(12.5) - R(12)$$

(c) The **equation**: Based on your answers to parts (a) and (b), which version of the linear approximation formula would fit this problem best, the "short form" or the "long form"? Write out this version of the linear approximation formula *using the variables in this problem* (instead of f and x).

$$R(12.5) - R(12) \approx R'(12)(12.5 - 12)$$

(d) What is the value of $R'(p)$ (that is, $\frac{dR}{dp}$) at the current ticket price of $p = 12$?

$$R'(p) = 3200 - 24p^2$$

$$R'(12) = -256$$

(e) Plug in the numbers from the other parts of this problem into your equation from part (c), and **solve**.

$$\Delta R \approx -256(0.5) = \$ -128$$